Fuzzy Splitting

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Fuzzy Logic (FL) is hot these days. Born in 1965 from the works of Professor Lofti Zadeh in the University of Berkeley, these theories have gained great acceptation nowadays, specially in Japan where they are applied in all sort of things from unmanned helicopters to videocameras. Probably you're not aware of it, but you use FL technologies everyday while driving your car, watching TV or even shaving yourself in the morning!. In this article, FL theories are introduced and applied towards an intelligent system in order to know how difficult to split is a given double star.

Everybody would agree that a double star whose components are separated by 10 seconds of arc is an open double, while another one whose components are only 1 second of arc apart is a tight double star. So far so good. Now, let me approach you while asking a simple, innocent at first question: "Where in the range from 1" to 10" would you change the adjective "tight" to "open"?

After a while, you probably answer me "Well, I'm not pretty sure, but maybe in an intermediate point, let's say, 5 seconds of arc".

Your reply is rather logical, but it creates a new, uncomfortable problem: from your classification, a double star such as Cp 13 (separation = 5.1") is an open double star, while Cor 148 (separation = 4.8") is a tight double. This sudden change of behavior doesn't fit well with the experience of double star observers. Wilhelm Struve, one of the greatest double stars observers from all times, suffered also from similar problems while making a classification for doubles in his Catalogus Novus (1827) depending on separation: Type I for doubles with less than 4" separation, Type II for doubles between 4" and 8", Type III between 8" and 16" and Type IV for separations between 16" and 32". He realized the problems arising for such a sharp classification and again divided Type I into Vicinae, Pervicinae and Vicinissimae. If you pay some attention to this sub-classification you will see that it puts the problem to sleep for a while, but it doesn't solve it.

Interestingly, we humans tend to communicate with other humans with linguistic terms that are not based directly on arithmetics, but in experience and knowledge. Imagine the following situation: Two buddies amateur astronomers are talking about their last double star observing session:

- "Joe, yesterday I started the session observing Epsilon Lyrae, the double double. It's a lovely system, you know, with those rather close components. Later I aimed the telescope to Otto Struve 298 in Bootes (separation = 1.0", same magnitude = 7.5 for both components). I suffered a lot until getting the split."
- "I know Mark, STF 298 is a really tight double!"

Joe and Mark don't even need to speak about numbers in order to express their observing experiences. Language allows them to exchange information easily, and terms like "rather close", "very tight" and so on are very useful when speaking about double stars, splittings and related terms. What a pity mathematics doesn't allow us to express such things, right?. Don't be worry, let's enter the beautiful paradise of Fuzzy Sets Theories (FST)!

The central idea about Fuzzy Sets Theories ("Fuzzy Logic" is the expression that usually gathers all things related to FST, from Fuzzy Control to Fuzzy Statistics) is based on the inexistence of the so named "law of excluded middles": From the times of Aristotle, we have grown in digital mathematical models: people is either tall or short, cars are fast or slow, things are cheap or expensive, everything is black or white. Nevertheless, the real world is made from different shades of gray!. Let me introduce you a Fuzzy Classification for separation in double stars. Pay a bit of attention now because you will hear some new powerful concepts. I promise everything will be easy to understand. It's only a question of a few minutes!

Fuzzyfication (what a word!)

Figure #1 represents a Linguistic Variable that express the concept "Separation in double stars". A Linguistic Variable is simply a collection containing several Fuzzy Sets. Here the used Fuzzy Sets are "Very Tight", "Rather Tight", " Tight", "Open Tight" and "Bit Open". In order to see what a Fuzzy Set is, let's take as an example the expression "Very Tight".

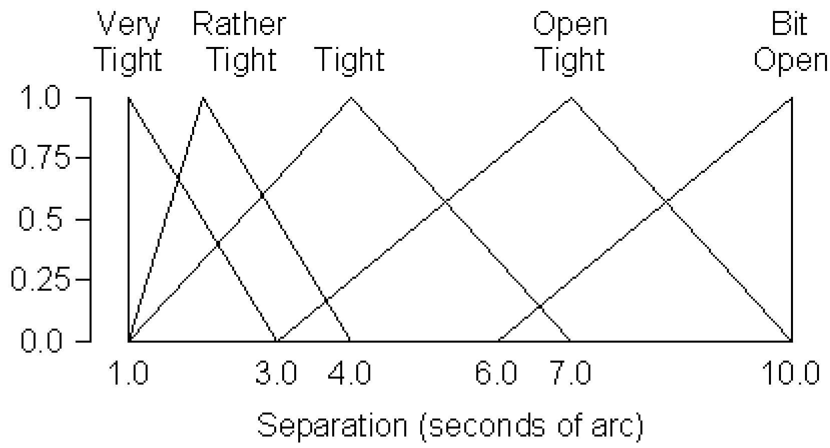

Figure #1: Linguistic Variable for concept "Separation"

As you can see, there is a triangle located at left in the figure with the label "Very tight" over it. The base of this triangle goes from 1.0 to 3.0 seconds of arc, so instinctively, we can say we have defined "Very tight" as every double star whose separation between components is bounded between 1 and 3 seconds of arc. Now the fun begins: Observe that the upper vertex of the triangle is located just over a vertical line from 1.0 seconds of arc. In FST, we say that the membership degree to the Fuzzy Set of "Very Tight" double stars is highest when separation equals 1.0 seconds of arc. Then, this membership degree starts to decline, let's say walking down along the triangle's hypotenuse, until reaching a membership degree of 0.0 when it reaches a separation of 3.0 seconds of arc. That's the real core of Fuzzy-Logic: Everything is a question of grade!

See now the representation of the Fuzzy Set "Rather Tight"; it starts at 1.0", reaches its maximum membership degree at 2.0" and ends at 4.0". The rest of Fuzzy Sets from the Linguistic Variable "Separation" are described in a similar way. In order to fix our ideas and also to experiment a bit with them, let's see how these concepts describe a "real" double star. Let's use Struve 3050 for it.

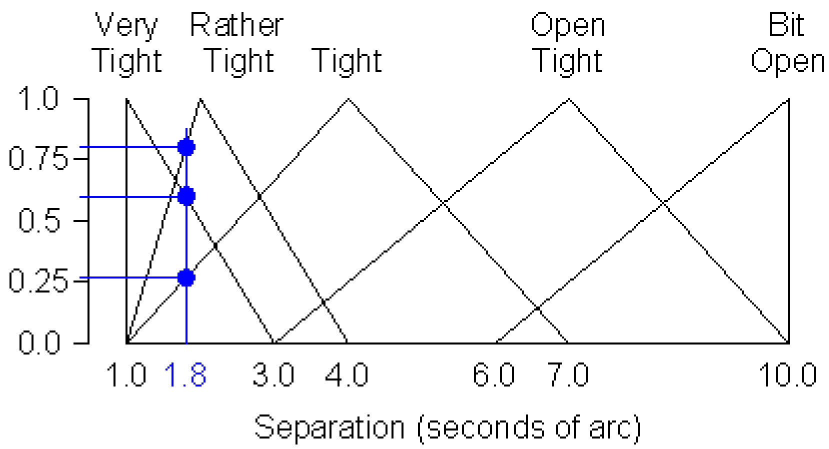

Figure #2: Membership degrees for STF 3050 with respect to Separation

This really nice double in Andromeda has a separation between components of 1.8 seconds of arc. As you can see from Figure #2, it has the following membership degrees:

- 0.8 to the Fuzzy Set of “Rather Tight” doubles
- 0.6 to the Fuzzy Set of “Very Tight” doubles
- 0.26 to the Fuzzy Set of “Tight” doubles

Using this approach, we can smoothly describe the concept Separation without sudden changes in classification. Also, under this model, Struve 3050 is a “Tight double”, a “Rather Tight” double and a “Very Tight” double star at the same time, although with different membership degrees. We can see it also as a way to express the feeling of observers using telescopes with different apertures. For a 2.5” aperture refractor, Struve 3050 is a very tight double, while the owner of a 8” SCT will observe it as a tight double, but not specially a very difficult one to split.

In order to evaluate difficulty in splitting a double star, observers also take into account the bright of the main and secondary components. This plays a crucial role for knowing in advance how difficult to split a star will be. Using the same strategies as above, we can easily build another Linguistic Variable for the concept “Difference of Magnitude”. Please, observe Figure #3 for a representation of this. I suspect you’re starting to enjoy this game!

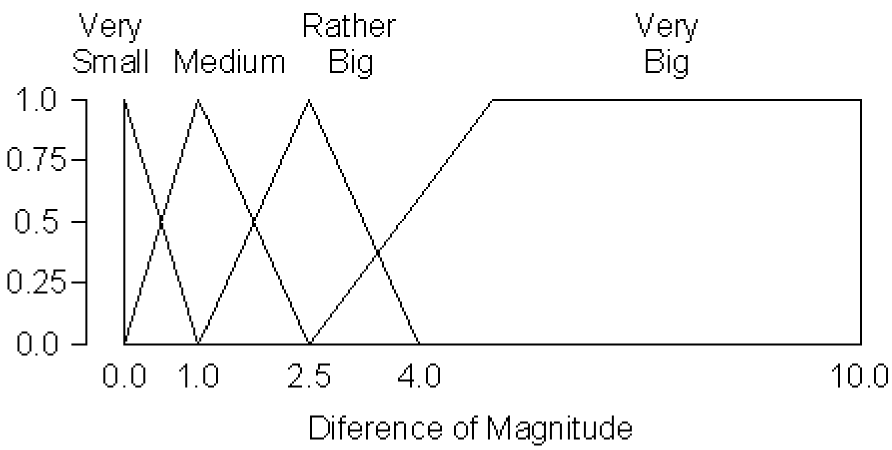

Figure #3: Linguistic Variable for concept "Difference of Magnitude"

Rules, rules everywhere!

If we ask Joe and Mark how they know in advance about the difficulty in splitting a double star, they will tell us some expert rules to apply. For example, they can tell us the following:

“If the separation between components is rather tight and the difference of magnitude is medium, then the double will be rather difficult to split”

Since Joe and Mark are lovely people with lots of patience, they agree to write for us a table describing all the possible combinations from our Linguístic Variables for Separation and Difference of Magnitude. Separations are expressed in columns, while rows represent Difference of Magnitude. The table looks as follows:

	Very Tight	Rather Tight	Tight	Open Tight	Bit Open
Very Small	Rather Difficult	Something Difficult	Something Easy	Rather Easy	Very Easy
Medium	Very Difficult	Rather Difficult	Something Difficult	Something Easy	Rather Easy
Rather Big	Very Difficult	Very Difficult	Rather Difficult	Something Difficult	Something Easy
Very Big	Very Difficult	Very Difficult	Very Difficult	Rather Difficult	Something Difficult

Table 1a : Expert Rules

As you can see, we already have Fuzzy Sets describing Separation and Difference of Magnitude, but nothing has been said yet about difficulty. Well, we could model a new Linguistic Variable describing a Difficulty Index (DI), going for example from 0.0 to 100.0, where a DI of 0 means a double splittable by virtually every scope, while a DI of 100.0 would imply the use of a good telescope under very good "seeing" skies. From the table given from Joe and Mark, we could directly label the Fuzzy Sets for Difficulty as "Very Easy", "Rather Easy", "Something Easy", "Something Difficult", Rather Difficult" and "Very Difficult". Figure #4 shows a representation for the concept "Difficulty Index".

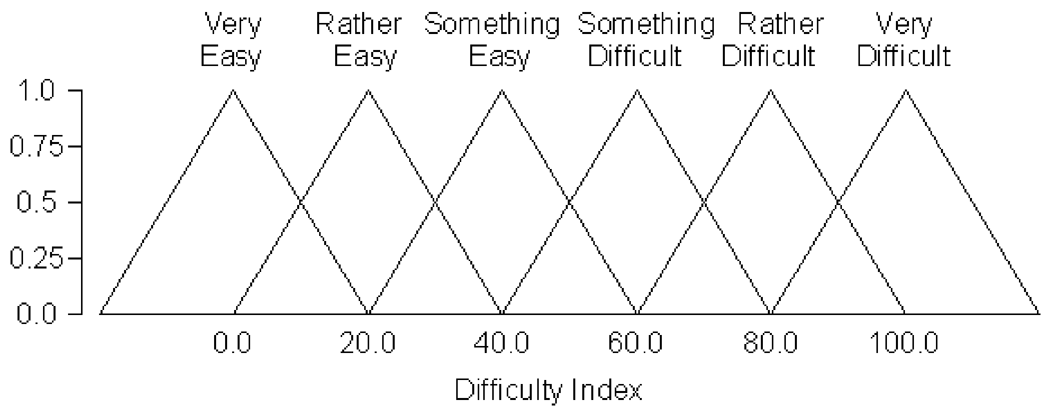

Figure #4: Linguistic Variable for concept "Difficulty"

Inferencing

This phase of the process is where a Fuzzy Logic based system "thinks". We could write some PhD level equations here, but suspect everything will be easier if we use an example close to our observing experiences. Let's take Algieba, Gamma Leonis for this. What?, you're a bit tired?. Ok, let's relax ourselves for a while and read a fine observing report of this pair. A member of the "Spirit of 33", Brendan Shaw, using a 5" fluorite refractor wrote the following log in Spring, 2000:

"A splendid double. Orange-red to my eyes. At X149 I felt I could drive a bus between the two stars. In the fleeting moments of good seeing there was clear black space between the pair. At x86 a playing card would have separated them, at x55 a piece of paper.

At the lower two powers the best view came when the clouds covered the stars! The lack of glare from the brighter star and the lack of diffraction rings meant there were two very clearly defined and separate pinpoints, even at X55. As the clouds thinned the stars tended to run into one another at the lower power. Not merging but more of a figure-8 and if I'd been sweeping quickly through the field I could have overlooked the pair. At X149 a delightful double regardless of the clouds. If only one of the components had been blue (one of the few other colours my weedy male eyes can distinguish) ... if anybody is starting a best-33-doubles in the northern hemisphere, Algieba gets a vote from me"

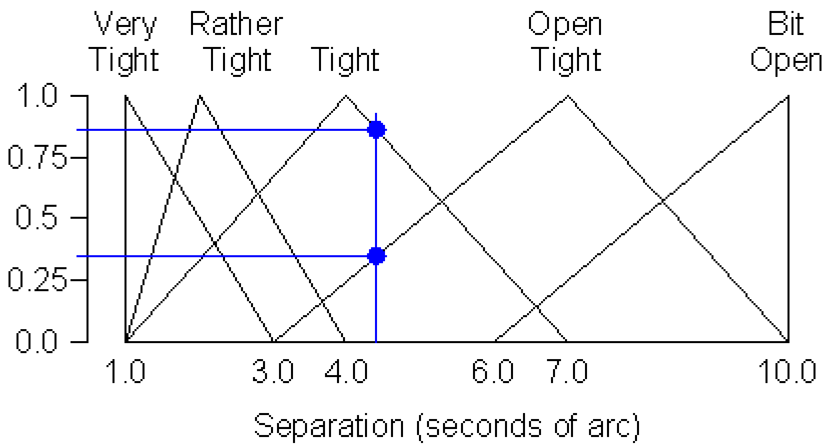

Figure #5: Membership degrees for Algieba with respect to Separation

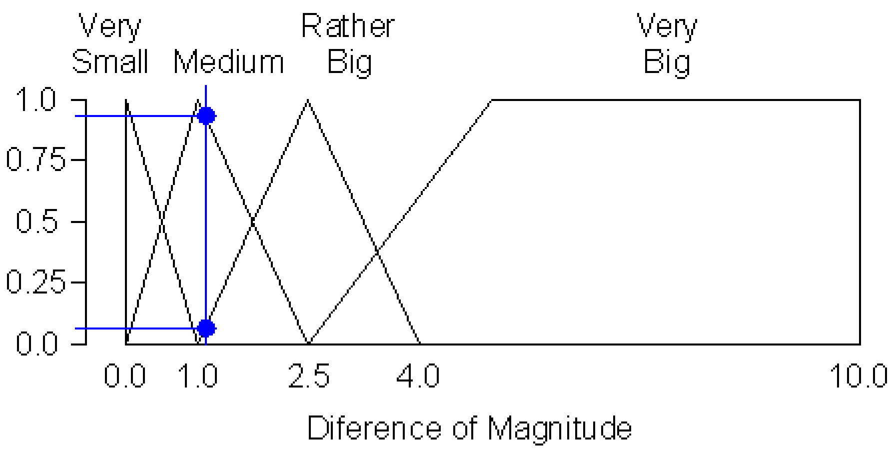

Figure #6: Membership degrees for Algieba with respect to Difference of Magnitude

Ok, let's return at class!. Algieba has its components separated by 4.4 seconds of arc, and they shine at 2.5 and 3.6 magnitude respectively, so the difference of magnitude is 1.1. Figures #5 and #6 show how to obtain the membership values for "Separation" and "Difference of Magnitude". These values are as follows:

Membership degrees for Separation:
Tight: 0.87
Open-Tight: 0.35

Membership degrees for Difference of Magnitude:
Medium: 0.93
Rather Big: 0.07

So, as described by our model, Algieba is a "Tight" and "Open-Tight" double (separation) with a "Medium" and "Rather Big" difference of magnitude. Now, what rule from the rules expressed in Table 1a must we apply?. Since we have "fired" two Fuzzy Sets for every Linguistic Variable involved, we need to get all possible combinations:

- Distance is Tight and Delta Magnitude is Medium
- Distance is Open-Tight and Delta Magnitude is Medium
- Distance is Tight and Delta Magnitude is Rather Big
- Distance is Open Tight and Delta Magnitude is Rather Big

Visually, we can see on Table 1b that in order to evaluate the Difficulty Index for Algieba, four rules have been triggered. This is another crucial point in Fuzzy Logic techniques: While traditional, old Intelligent Systems fired only one rule at a time, Fuzzy Logic works in parallel processing mode!

	Very Tight	Rather Tight	Tight	Open Tight	Bit Open
Very Small	Rather Difficult	Something Difficult	Something Easy	Rather Easy	Very Easy
Medium	Very Difficult	Rather Difficult	Something Difficult	Something Easy	Rather Easy
Rather Big	Very Difficult	Very Difficult	Rather Difficult	Something Difficult	Something Easy
Very Big	Very Difficult	Very Difficult	Very Difficult	Rather Difficult	Something Difficult

Table 1b : Rules Fired for evaluating Algieba

Now, another question arises: what membership degree will be applied to the Fuzzy Sets "Something Easy", "Something Difficult" and "Rather Difficult" describing the Difficulty Index?. Well, Fuzzy Logic offers the modeler several alternatives for describing a "Fuzzy-AND" operator. Nevertheless, the most used one is the "minimum value". Let's see this for the first rule for Algieba:

If Distance is **Tight** and Delta Magnitude is **Medium** then Difficulty is Something Difficult

Now, let's substitute the Fuzzy Sets in bold font by their membership degrees:

If Distance is **0.87** and Delta Magnitude is **0.93** then Difficulty is Something Difficult

Here, the minimum value of 0.87 and 0.93 is 0.87, so, using something named "Modus Ponens" in Logic, the resulting membership value for the Fuzzy Set "Something Difficult" will be 0.87. In another words, for Algieba, the first rule becomes:

If Distance is **Tight (<u>0.87</u>)** and Delta Magnitude is **Medium (0.93)** then Difficulty is Something Difficult **(<u>0.87</u>)**

Doing the same for every rule affected for Algieba, we would obtain Table 1c:

	Very Tight	Rather Tight	Tight	Open Tight	Bit Open
Very Small	Rather Difficult	Something Difficult	Something Easy	Rather Easy	Very Easy
Medium	Very Difficult	Rather Difficult	Something Difficult (0.87)	Something Easy (0.35)	Rather Easy
Rather Big	Very Difficult	Very Difficult	Rather Difficult (0.07)	Something Difficult (0.07)	Something Easy
Very Big	Very Difficult	Very Difficult	Very Difficult	Rather Difficult	Something Difficult

Table 1c : Some Rules get more "weight" than others!

That is, Algieba has a Difficulty Index expressed with the following membership degrees:

- Something Easy: 0.35
- Something Difficult: 0.87
- Something Difficult: 0.07
- Rather Difficult: 0.07

You can see this as a board of four members from a committee of experts in double stars (Joe and Mark are included, of course) evaluating the Difficulty of splitting Algieba. The first member of them says it is "something easy" to split, and has a "confidence degree" in his vote of 0.35. In the same way, every expert expresses his/her opinion and gives also a "confidence factor". As you can see, Fuzzy Logic is a true democratic system, but newspapers need a verdict for Algieba!. Fortunately, the board of experts have some tricks to elaborate a final result, that is, a Difficulty Index (**DI**) between 0 and 100.

Defuzzyfying processes

For coming back from the Fuzzy world to the "normal" world, we need to convert a result expressed as a collection of membership degrees to a numerical result. After all, almost every Fuzzy system needs a real value as an output-result (for example, the number of grams/minute of fuel required by your car's engine is a mensurable quantity, although it has been previously calculated by a fuzzy system if your car has been manufactured in Japan).

In order to accomplish this, there are several techniques available to fuzzy designers. I even developed one in 1998 and I was very happy with my research until speaking one day with Professor Michio Sugeno from Tokyo Institute of Technology at my office, realizing he developed it 10 years before :). Ok, let's continue; The important concept is that we can associate the resulting membership degrees for difficulty as "weights" locating these "weights" in the basis' central point of every triangle representing the respective Fuzzy-Set. The following Figure (#7) will make things crystal clear:

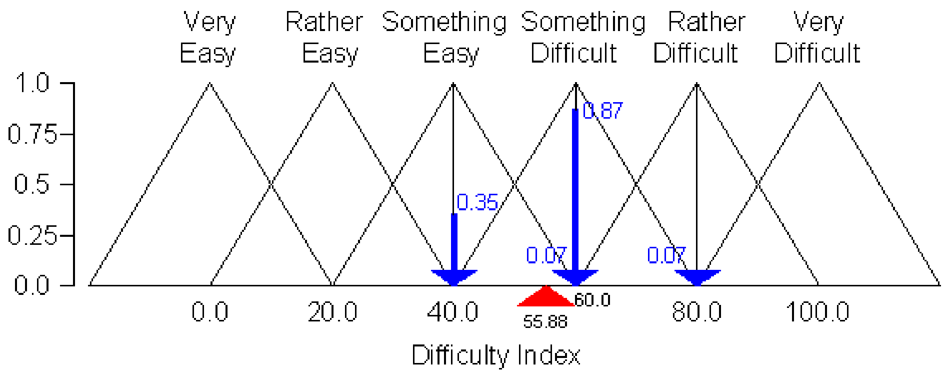

Figure #7: Defuzzyfication Procedure

You can see the weights represented by vertical arrows, with longitudes expressing how much "weight" (membership degree) is associated. Now, what if we look for a point on the horizontal axis that would equilibrate these weights??. Yes, my friend, that's the output for a Fuzzy-System using Sugeno's "Singletons" modelling and it's represented in the Figure by the a red arrow pointing upwards. This method of defuzzyfication has the advantages of being intuitive and easy to calculate, so it's very fast in critical control systems. The formal expression for this defuzzyfication method is as follows:

$$x = \Sigma(w_i \, . \, x_i) / \; \Sigma(w_i)$$

where x is the final result (Difficulty Index), wi are the weights (output membership degrees) and xi are the points (horizontal axis) where the weights are applicated. Substituting the values from Algieba in this formula, we will have:

DI = (0.35 x 40) + (0.87 x 60) + (0.07 * 60) + (0.07 * 80) / (0.35 + 0.87 + 0.07 + 0.07)

So,

DI = 55.88

That is the obtained Difficulty Index obtained for Gamma Leonis. As you can see, all the process requires a lot of computation if you try to make them by hand. Fortunately there are computers!

LADIC comes to rescue!

After building a program for designing and running Fuzzy systems and testing them, designers usually create what is called a "Control Surface". Basically, it's a sampling of the variables involved (here, "Separation" and "Difference of Magnitude" are the variables) relating the output-results for every possible combination of variables' values. It has the great advantage of being easily portable and compact and it's almost child's play for an electrical engineer to put these values into an electronic chip. I decided to create a Control Surface for the **DI** system in double stars dividing the ranges of separation and difference of magnitude in 100 parts. This generated a grid of 10,000 points and then a **DI** was calculated for every point, so soon a collection of 10,000 3D points was created (it took about 3 minutes of number-crunching to my old computer).

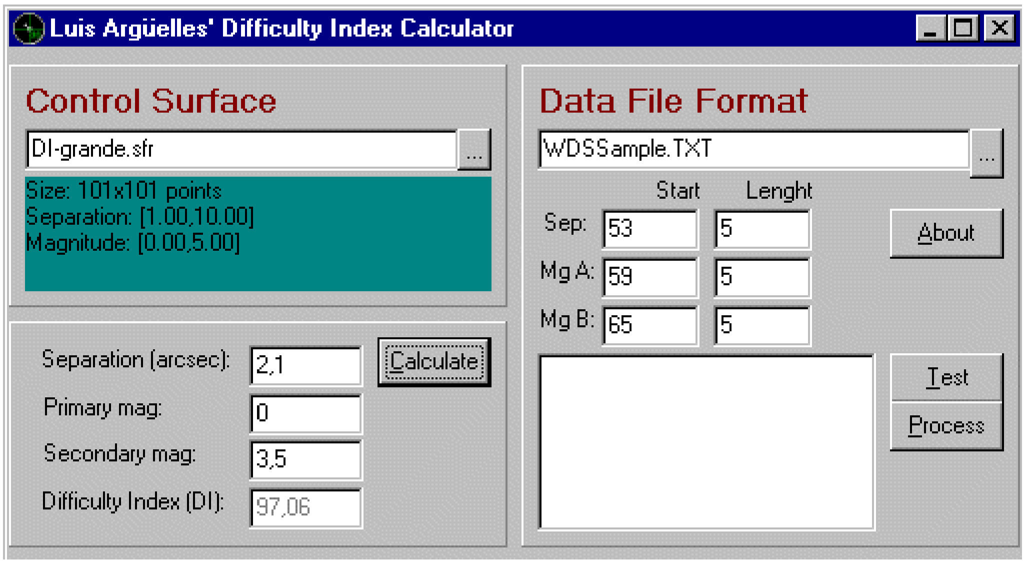

Figure #8: LADIC program

The rest was easy. Rafael Barberá, another member of the ***Spirit of 33***, took the Control Surface and wrote a program to read the Surface and interpolate those values not falling exactly in the grid. The result is a program named LADIC.

DI Interpretation

After reading all the above, you look seriously at me, and say: "Ok, theory sounds good, but in the real world I own a 6 inches aperture reflector. What's the reachable **DI** with my scope?". Well, before replying to your question, we must deeply realize that a **DI** is no more and no less than a figure expressing and describing **inherent features** of a given double star. From this, we will establish a relationship between DI and not only aperture, but also optical quality of your scope, mean seeing conditions from your observing site and even your experience as a double star observer!

We simply can not say: The **DI** for a 4" refractor is x, because it will depend on the glass quality of that given 4" (optics quality while observing double stars is almost as important as aperture), your local seeing (a 4" refractor user located in Atacama, Chile, will reach a different **DI** that another observer located, let's say, in NY City) and the observer's experience: Given the same scope, observers such as Couteau, Comellas, Susan French or Sissy Haas, to name only a few, will reach a superior **DI** than you and me. The key concept here is that you must calibrate your observing system (telescope plus local seeing plus observing experience) towards **DI** values. Don't be afraid. This is a really easy process.

Use LADIC or the tables from the observing programs in this site for choosing some doubles with a **DI** between 75 and 100. Then, observe your particular calibration set of doubles in a standard "seeing" night, that is, avoid those nights with poor seeing, but also forget about calibrating for **DI** in those rare nights when you can split everything!. After splitting a double from your set, don’t forget to log the name of the double and its associated **DI**. After some observing sessions take the 5 or 10 highest **DI**s reached by your observing system and calculate the mean. This value will be your reference, and from then, **DI** values will serve you as the best evaluation system for knowing in advance how difficult to split will be any given double star.

Finally, I would like to express my deepest gratitude to Rafael Barberá for not only writing a superb interpolation program (it is able to process the complete Washington Double Star Catalogue in less than 7 seconds), but also for encourage me to write this article.

Important notes:

- If you input Algieba values into LADIC, you'll get a **DI** of 56.15 instead 55.88. Please, have into account that LADIC is an interpolation program, and there is a very small (about 0.5%) difference towards the exposed, analytical result. Since my LISP compiler generates an executable program of about 15 Mbytes for evaluating **DI**, the very small loss in precision is insignificant and more than compensates the download time it would require to use the "pure" system.

- You can download LADIC and use it freely from the following address:
http://whuyss.tripod.com/Fuzzy-splitting/fuzzy-splitting.html